DIAS **Delhi Institute for Administrative Services**
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ALL INDIA TEST SERIES CSE-2023

Candidate 's Information

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4. SUBJECT:- ..... Mechanics .....
5. DATE:- ..... 23 July 2023 .....

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Q.NO	MARKS
1.	30
2.	31
3.	
4.	27
5.	30
6.	22
7.	
8.	

Very good keep it up!

TOTAL MARKS	140

252

EXAMINER SIGNATURE

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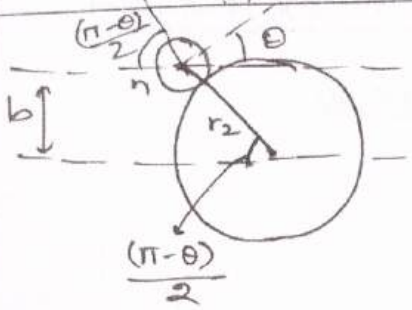
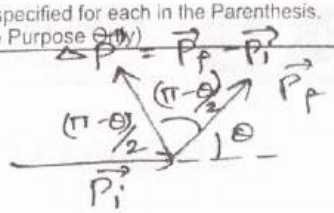
1(a)





$$b = (r_1 + r_2)\sin\left(\frac{\pi-\theta}{2}\right) = (r_1 + r_2)\cos\frac{\theta}{2}$$

Impact parameter $= R\cos\theta/2$

differential cross section in terms of impact parameter –

$$\sigma(\theta) = -\frac{b}{\sin\theta}\frac{db}{d\theta}$$

$$\boxed{\sigma(\theta) = \frac{R^2}{4} = \frac{(r_1+r_2)^2}{4}}$$

Total cross-section is given as –

$$\sigma = \int_0^{\pi} \sigma(\theta)\, 2\pi \sin\theta\, d\theta$$

$$\boxed{\sigma = \pi(r_1+r_2)^2}$$ which is the effective area of cross section, the incident particle faces.

6/10

Q 6 b) Equation of orbit under central force is given as

$$\boxed{\frac{d^2u}{d\theta^2} + u = -\frac{mF(u)}{J^2u^2}}$$

where $u = \frac{1}{r}$ (r is path of particle)

Given $u = \frac{1}{r} = \frac{1}{a(1+\cos\theta)} = \frac{\sec^2\theta/2}{2a}$

$$\frac{du}{d\theta} = \frac{1}{2a}\cdot \sec^2\theta/2 \tan\theta/2$$

$$\frac{d^2u}{d\theta^2} = \frac{1}{2a}\sec^2\theta/2 \tan^2\theta/2 + \frac{1}{2a}\sec^4\theta/2$$

$$= \frac{1}{2a}\left[\sec^4\theta/2 - \sec^2\theta/2\right] + \frac{1}{2a}\sec^4\theta/2$$

$$= \frac{1}{a}\sec^4\theta/2 - \frac{1}{2a}\sec^2\theta/2$$

$$\boxed{\frac{d^2u}{d\theta^2} = 4au^2 - u}$$

putting in equation

$$(4au^2 - u) + u = -\frac{mF(u)}{J^2u^2}$$

$$-\frac{4au^2 \cdot J^2u^2}{m} = F(u)$$

$$\Rightarrow \boxed{F(u) = -\frac{4aJ^2}{m}u^4}$$

$$\Rightarrow \boxed{F(r) \propto \frac{1}{r^4}}$$

inverse 4th power of r.

6/10

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1 c) Before collison – Total Energy =

$E = 2m_0c^2$ $\quad m_0c^2$

$$E = 2m_0c^2 + m_0c^2$$

$$\boxed{E_i = 3m_0c^2}$$

⇓

M, v (After collision)

Conservation of energy gives –

$$2m_0c^2 + m_0c^2 = Mc^2$$

$$\Rightarrow \boxed{M = 3m_0}$$ moving mass of composite particle.

Momentum of 1st (incident) particle –

$$p^2c^2 = E^2 - (m_0c^2)^2$$

$$\Rightarrow p = \sqrt{3}\, m_0 c$$

6/10

By momentum conservation –

$$\sqrt{3}\, m_0 c = Mv$$

$$\Rightarrow v = \frac{\sqrt{3}\, m_0 c}{3m_0} = \frac{c}{\sqrt{3}} = \boxed{1.73 \times 10^8 \text{ ms}^{-1}}$$

Rest mass of composite particle –

$$M_0 = M\left(\sqrt{1 - \frac{v^2}{c^2}}\right) = 3m_0\sqrt{1 - \frac{1}{3}} = \sqrt{6}\, m_0$$

$$\boxed{M_0 = 2.449\, m_0}$$

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1(d) Power is work done per unit time.

$$\text{Power} = \frac{\text{Work}}{\text{time}} = \frac{T\,d\theta}{dt} = T\omega$$

(T: Torque.)

$$75 \times 10^3 = T \times \omega$$

$\omega = 1800$ rev/min

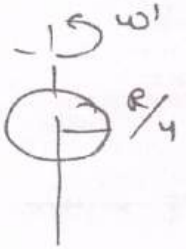
$$= \frac{1800 \times 2\pi}{60} \frac{\text{radian}}{\text{sec.}}$$

$\omega = 60\pi$ radian/sec

$$\Rightarrow T = \frac{75 \times 10^3}{60\pi} = 397.88 \text{ Nm}$$

6/10

1(e) Conservation of angular momentum —



$$mR^2\omega = m\left(\frac{R}{4}\right)^2\omega'$$

[mass remains same since contraction]

$$\Rightarrow \boxed{\omega' = 16\omega}$$

length of day depend on ω as

$$T = \frac{2\pi}{\omega}$$

Thus $T' = \frac{2\pi}{\omega'}$,

$$\Rightarrow \frac{T'}{T} = \frac{\omega}{\omega'}$$

$$\Rightarrow T' = \frac{\omega}{\omega'}T = \frac{T}{16}$$

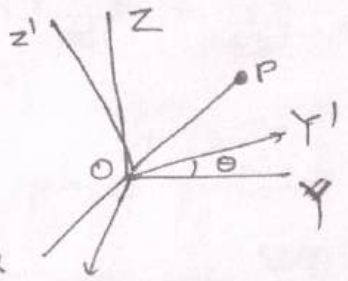
6/10

Thus length of day will reduce by 1/16 th factor. ⇒ 24 will become-

$$T' = \frac{24}{16} = \frac{3}{2} = 1.5 \text{ hours}$$

Q.2(a) Rotating frame of reference in a Non-inertial frame thereby experiences fictious forces or pseudo forces.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= x'\hat{i}' + y'\hat{j}' + z'\hat{k}'$$



$$\frac{d\vec{r}}{dt} = \frac{dx'}{dt}\hat{i}' + \frac{dy'}{dt}\hat{j}' + \frac{dz'}{dt}\hat{k}' + x'\frac{d\hat{i}'}{dt} + y'\frac{d\hat{j}'}{dt} + \frac{d}{dt}\hat{k}'$$

$$\vec{V} = \vec{\omega} \times \vec{r}$$

$$\because \frac{dr}{dt} = \omega \times r$$

$$\Rightarrow \frac{di}{dt} = \omega \times \vec{i}$$

$$\frac{d\vec{r}}{dt} = \frac{d'\vec{r}'}{dt} + \omega \times (x'\hat{i}' + y'\hat{j}' + z'\hat{k}')$$

$$\boxed{\frac{d\vec{r}}{dt} = \frac{d'\vec{r}'}{dt} + \omega \times \vec{r}} \qquad \boxed{\because \vec{r}' = \vec{r}}$$

This leads to **transport theorem** –

$$\left[\frac{d}{dt}(\,)\right]_{\substack{\text{space}\\ \text{set}\\ \text{of}\\ \text{frame}}} = \left[\frac{d'}{dt}(\,) + \vec{\omega} \times (\,)\right]_{\substack{\text{Body}\\ \text{frame}}}$$

putting $\vec{V}$, to get equation of motion.

$$\frac{d}{dt}\vec{v} = \frac{d'}{dt}\vec{v} + \vec{\omega} \times \vec{v}$$

We have $\boxed{\vec{v} = \vec{v}' + \vec{\omega} \times \vec{r}}$

$$\Rightarrow \frac{d}{dt}\vec{v} = \frac{d'}{dt}(\vec{v}' + \vec{\omega} \times \vec{r}) + \vec{\omega} \times (\vec{v}' + \vec{\omega} \times \vec{r})$$

$$= \frac{d\vec{v}}{dt} = \frac{d'\vec{v}'}{dt} + \frac{d'\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d'\vec{r}}{dt} + \vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

for rotating body with uniform ω

$$\Rightarrow \frac{d'\vec{\omega}}{dt} = 0$$

$$\Rightarrow \vec{a} = \vec{a}' + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

or $\boxed{\vec{a}' = \vec{a} - 2\vec{\omega} \times \vec{v}' - \vec{\omega} \times (\vec{\omega} \times \vec{r})}$

$-2\vec{\omega} \times \vec{v}'$: coriolis force accelerat

$-\vec{\omega} \times (\vec{\omega} \times \vec{r})$: centrifugal force accelerat

force ie —

$$\boxed{m\vec{a}' = m\vec{a} - 2m(\vec{\omega} \times \vec{v}') - m\vec{\omega} \times (\vec{\omega} \times \vec{r})}$$

Equation of motion in rotating frame.

Calculation of deflection suffered by free falling body —

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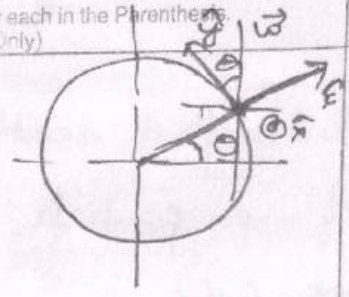

$\vec{\omega} = \omega\cos\theta\,\hat{j} + \omega\sin\theta\,\hat{k}$

$\vec{v}' = -v\,\hat{x}$

Coriolis force experienced

$= -2m(\vec{\omega} \times \vec{v}')$

$$\begin{vmatrix} i & j & k \\ 0 & \omega\cos\theta & \omega\sin\theta \\ 0 & 0 & -v \end{vmatrix}$$

$= -2m(-v\omega\cos\theta)$

$\Rightarrow$ $\boxed{F_{coriolis} = 2mv\omega\cos\theta\,\hat{i}}$ along x-axis

$m\frac{d^2x}{dt^2} = 2m = 2mg t\omega\cos\theta$

as $v = gt$

$\Rightarrow \frac{d^2x}{dt^2} = 2gt\omega\cos\theta$

$\frac{dx}{dt} = gt^2\omega\cos\theta \quad [\text{at } t=0,\ x=0]$

$\Rightarrow x = \int gt^2\omega\cos\theta = \frac{gt^3}{3}\omega\cos\theta$

Since $t = \sqrt{\frac{2H}{g}}$

$\Rightarrow \boxed{x = \frac{g}{3}\left(\frac{2H}{g}\right)^{3/2}\omega\cos\theta}$

B/W

deflection suffered by it in East direction.

In Northern hemisphere – Rightward deflect^n

In Southern hemisphere – leftward

→ reason for cyclones

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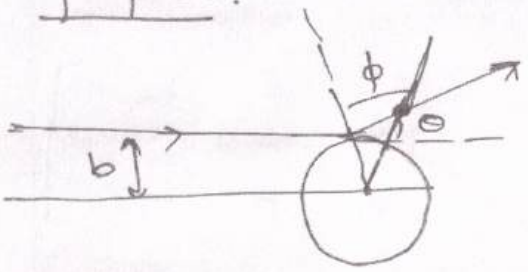
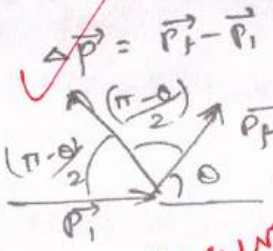
2(b) Rutherford scattering is scattering of α-particle by heavy nucleus eg. Gold.





Assumptions ?

Momentum is given as –

$$\int \vec{F}.dt = \Delta\vec{p} = \vec{P}_f - \vec{P}_i = 2\vec{p}_i \sin\frac{\theta}{2}$$

$$\int_{-(\pi-\theta)/2}^{(\pi-\theta)/2} \frac{k}{r^2}\cdot\cos\phi\,.d\phi\,.\frac{dt}{d\phi} = 2\vec{p}_i \sin\frac{\theta}{2} \qquad \vec{F} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r^2} = \frac{k}{r^2}$$

= from momentum conservation –

$$p_i b = m r^2 \omega \qquad [\text{also } p_i = \sqrt{2mE_\alpha}]$$

$$\Rightarrow \int_{-(\pi-\theta)/2}^{(\pi-\theta)/2} \frac{k}{r^2}\frac{\cos\phi}{\omega}\cdot d\phi = 2p_i \sin\theta/2$$

$$\int_{-(\pi-\theta/2)}^{(\pi-\theta/2)} \frac{mk\cos\phi\, d\phi}{p_i b} = 2p_i \sin\theta/2$$

$$\Rightarrow \boxed{b = \frac{k}{2E_\alpha}\cot\theta/2}$$

Impact parameter.

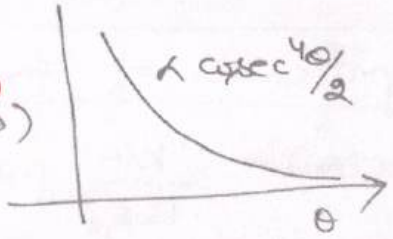
E_α = K.E. of α particle.

~~Total~~ differential scattering cross-section is –

$$\sigma(\theta) = \frac{-b}{\sin\theta}\frac{db}{d\theta} \qquad b = \frac{K}{2E_\alpha}\cot\theta/2$$

$$\frac{db}{d\theta} = \frac{K}{2E_\alpha}\operatorname{cosec}^2\theta/2 \cdot \frac{1}{2}$$

$$\boxed{\sigma(\theta) = \frac{K^2}{16E_\alpha^2}\operatorname{cosec}^4\theta/2}$$



Total cross section – $\sigma(\theta)$

$$\sigma = \int_0^\pi \sigma(\theta)\, 2\pi\sin\theta\, d\theta$$

$$= \frac{K^2}{16E_\alpha^2}\cdot 2\pi \cdot \int_0^\pi \operatorname{cosec}^4\theta/2 \cdot \sin\theta \cdot d\theta$$

$\sigma \longrightarrow \infty$ turns out to be infinite.

12/20

Reason –

① Assumption of infinite range of force as $F = K/r^2$ but due to [e⁻ screening] the range remains finite.

② Particles are assumed to be

point particles. Their finite size is ignored.

(5) Collision is taken as Elastic between incident and target particle.

Significance of formula

$$\sigma(\theta) = \frac{k^2}{16 E_\alpha^2} \operatorname{cosec}^4 \theta/2$$

(1) Range of nuclear force can be known.

(2) Scattering cross section →

↳ (1) proportional to square of charges of incident & target

↳ (2) Inversely proportional to Squared Kinetic energy.

2(c) Young's Modulus

$$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta l}{L}\right)} = \frac{FL}{A\Delta l}$$

Work done $= \int F \cdot dl = \int_0^l \frac{YA l\, dl}{L}$

$$= \frac{YA l^2}{2L}\Big]_0^l = \frac{YA l^2}{2L}$$

Work done per unit volume $= \frac{Yl}{2L} \cdot \frac{l}{L}$

$$\Rightarrow \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \cdot \frac{F}{A} \cdot \frac{l}{L} \quad [\because V = AL]$$

$= \frac{1}{2}$ Stress x Strain

Work done $= \frac{YA l^2}{2L}$

$$= \frac{2 \times 10^{12} \times (10^{-2}) \times (0.01)^2}{2 \times 20} \text{ ergs.}$$

$$= 5 \times 10^{+3} \text{ ergs}$$

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Work = 5000 ergs

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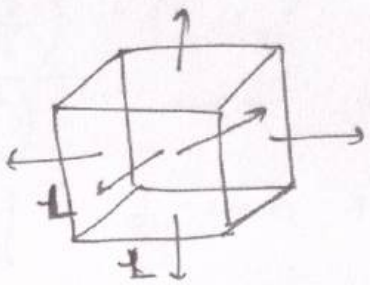
Q.49)

Let tensile force acting along all faces of cube — thus new dimensions of box will be —



$$L_x = L + \frac{PL}{Y} - \sigma\frac{PL}{Y} - \sigma\frac{PL}{Y} = L + \frac{PL}{Y}(1-2\sigma)$$

(longitudinal extension; lateral compression)

$$L_y = L + \frac{PL}{Y} - \sigma\frac{PL}{Y} - \sigma\frac{PL}{Y} = L + \frac{PL}{Y}(1-2\sigma)$$

$$L_z = L + \frac{PL}{Y} - \sigma\frac{PL}{Y} - \sigma\frac{PL}{Y} = L + \frac{PL}{Y}(1-2\sigma)$$

$$\Rightarrow V' = L_x L_y V_z$$

$$V' = L^3\left[1 + \frac{P}{Y}(1-2\sigma)\right]^3$$

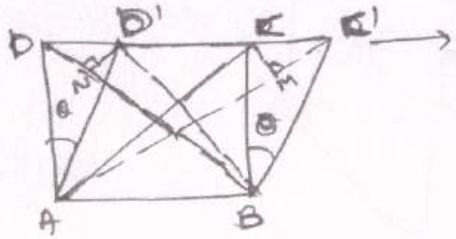
$$V' = L^3\left[1 + \frac{3P}{Y}(1-2\sigma)\right]$$

$$\frac{V'}{V} - 1 = \frac{3P}{Y}(1-2\sigma)$$

$$= \frac{P}{K} = \frac{3P}{Y}(1-2\sigma)$$

$$\Rightarrow \boxed{Y = 3K(1-2\sigma)}$$

Applying tangential force along one face of cube -



Due to stress, extension in AC

$$\Delta(AC) = \frac{P(AC)}{Y}$$

Sim. compression in BD =

$$\Delta BD = \frac{P(BD)}{Y}$$

$$\Delta AC = \frac{P(AC)}{Y} + \sigma\frac{P(AC)}{Y}$$

$$\Delta AC = CC'\cos 45^\circ = CD\theta\cos 45 = \frac{CD\theta}{\sqrt{2}}$$

$$\frac{CD\theta}{\sqrt{2}} = CD\sqrt{2}\cdot\frac{P}{Y}(1+\sigma)$$

$$\Rightarrow \frac{P}{\theta} = \eta \Rightarrow \boxed{Y = 2\eta(1+\sigma)}$$

(modulus of rigidity)

from above two relations — eliminating σ.

$$Y = 3K\left[1 - 2\left(\frac{Y}{2\eta} - 1\right)\right]$$

$$\frac{Y}{3K} = \left[1 - \frac{Y}{\eta} + 2\right]$$

$$\Rightarrow \frac{Y}{3K} = 3 - \frac{Y}{\eta}$$

$$\frac{1}{3K} = \frac{3}{Y} - \frac{1}{\eta}$$

$$\Rightarrow \frac{9}{Y} = \frac{1}{K} + \frac{3}{\eta}$$

$$\Rightarrow \boxed{Y = \frac{9K\eta}{3K + \eta}}$$

These relations prove the Y, η, K are dependent on each other.

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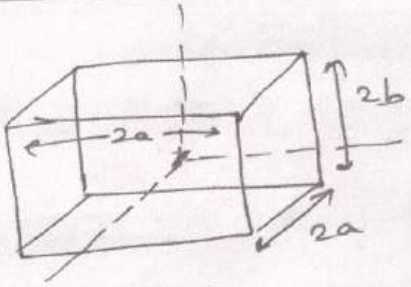
4(b)

Euler's equations of rotational motion are —



. rectangular parallelopiped.

$$N_1 = I_1\dot{\omega}_1 - (I_2 - I_3)\,\omega_2\omega_3$$

$$N_2 = I_2\dot{\omega}_2 - (I_3 - I_1)\,\omega_3\omega_1$$

$$N_3 = I_3\dot{\omega}_3 - (I_1 - I_2)\,\omega_1\omega_2$$

Since the parallelo-piped has

$I_1 = I_2 = I$ as symmetric about 3 axis.

$\Rightarrow$

$$N_3 = I_3\dot{\omega}_3 = 0 \quad \text{as No external torque.}$$

$\Rightarrow$ $\boxed{\omega_3 = \text{constant}}$

also $$I_2\dot{\omega}_2 = (I_3 - I_1)\,\omega_3\omega_1$$

$$\dot{\omega}_2 = \left(\frac{I_3 - I_1}{I_1}\right)\omega_3\omega_1 \qquad \text{Si'ce } I_2 = I_1$$

Sim. $$\dot{\omega}_1 = -\frac{(I_1 - I_3)}{I_1}\,\omega_3\omega_2$$

$\Rightarrow$ $\dot{\omega}_2 = -\Omega\omega_1$ where $\boxed{\Omega = \dfrac{I_3 - I_1}{I_1}\,\omega_3}$

$\dot{\omega}_1 = -\Omega\omega_2$

differentiating

$$\dot{\omega_2} = \Omega\omega_1 \quad \& \quad \dot{\omega_1} = -\Omega\dot{\omega_2}$$

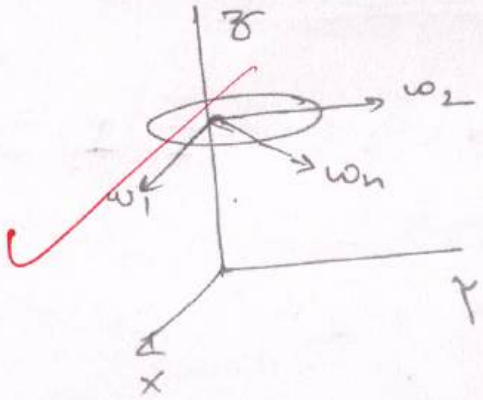
putting $\omega_1 = -\Omega\omega_2$

$$\Rightarrow \dot{\omega_2} = -\Omega^2\omega_2$$

$$\Rightarrow \dot{\omega_1} = -\Omega^2\omega_1$$

$$\Rightarrow \omega_1 = A\sin\Omega t$$

$$\omega_2 = A\cos\Omega t$$



traces out a circle as

$$\boxed{\omega_1^2 + \omega_2^2 = A^2}$$ circle equation.

with precessional frequency

$$\boxed{\Omega = \left(\frac{I_3 - I_1}{I_1}\right)\omega_3}$$

Thus the motion is periodic with per frequency Ω.

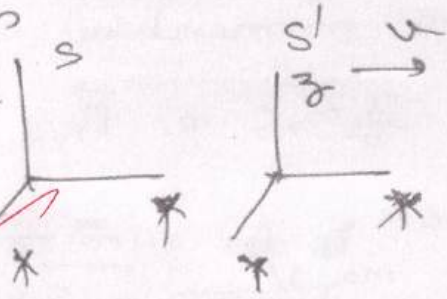
4(c)

Volume element

$v = dx\,dy\,dz\,dt$ is invariant.

from length contraction

$dx' = \gamma\, dx$ (proper length)



from time dilation —

$dt' = \frac{dt}{\gamma}$ proper time

where,

$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$

$dy' = dy$

$dz' = dz$

Lorentz transform

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

$\Rightarrow v' = dx'dy'dz'dt'$

$= \gamma\, dx\, dy\, dz\, \frac{dt}{\gamma} = dx\,dy\,dz\,dt$

$\Rightarrow v' = dx'dy'dz'dt' = dx\,dy\,dz\,dt = V$

thus it remains lorentz invariant.

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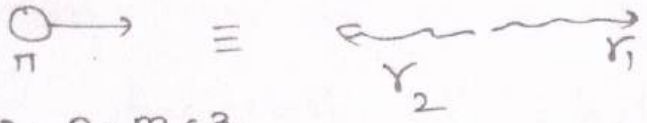
5(a)



m_0, $p = m_0 c \frac{3}{4}$

r_π

from momentum conservation —

$$\frac{3}{4} m_0 c = \vec{P}_{r_1} + \vec{P}_{r_2} \quad \text{——— (1)}$$

also for pion —

$$\frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \frac{3}{4} m_0 c$$

$$\Rightarrow \boxed{v = \frac{3}{5} c} \quad \text{and} \quad \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{5}{4}$$

for photons rest mass = 0

$$\Rightarrow E = pc \qquad (\because \vec{P}_{r_2} = -P \text{ as opposite direction})$$

$$\Rightarrow \frac{3}{4} m_0 c^2 = E_1 - E_2 \quad \text{——— (2)}$$

also energy conservation —

$$\frac{5}{4} m_0 c^2 = E_1 + E_2 \quad \text{——— (3)}$$

from (2) & (3)

$$2E_1 = \frac{8}{4} m_0 c^2 \Rightarrow E_1 = m_0 c^2$$

$$2E_2 = \frac{2}{4} m_0 c^2 \Rightarrow E_2 = \frac{m_0 c^2}{4}$$

6/10

5(b) Earth orbits in elliptical path around Sun.

with $K.E. = \frac{1}{2}mv^2 = \frac{GMm}{2r}$

Potential energy $\left(\text{as } \frac{mv^2}{r} = \frac{GMm}{r^2}\right)$

$P.E. = -\frac{GMm}{r}$

Total Energy $T.E. = -\frac{GMm}{2r}$

velocity increased by $\frac{1}{2}$ ⇒

$v_f = v + \frac{v}{2} = \frac{3v}{2}$

for parabolic orbit $e = 1$

& Energy $= 0$

Thus new K.E. will be –

$\frac{1}{2}mv_f^2 = \frac{1}{2}m\cdot\left(\frac{3v}{2}\right)^2 = \frac{9}{4}K.E.$

$= \frac{9}{4}\left(\frac{GMm}{2r}\right)$

$E_{total} = \frac{9}{4}\frac{GMm}{2r} - \frac{GMm}{2r}$

6/10

5(c)

Angular momentum

$\boxed{J = I\omega}$ Earth solid sphere

$\Rightarrow I = \frac{2}{5} m R^2$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 60 \times 60}$

$\Rightarrow J = \frac{2}{5} \times 6 \times 10^{24} \times (6.4)^2 \times (10^6)^2 \times \frac{2\pi}{24 \times 60 \times 60}$

$\boxed{J = 7.145 \times 10^{33}}$ $Kg\ m^2\ sec^{-1}$

Rotational kinetic energy –

$K.E. = \frac{1}{2} I \omega^2$ I of earth $= 9.83 \times 10^{37}$

$\frac{1}{2} \times 9.83 \times 10^{37} \times (7.27 \times 10^{-5})^2$ $\omega = 7.27 \times 10^{-5}$

$\boxed{K.E. = 2.59 \times 10^{29}\ Joule.}$

$(3.5 \times 10^9 \times 10^3 J_s)\ t = 2.59 \times 10^{29}$

$\Rightarrow t = 7.4 \times 10^{16}$ seconds

$= 8.56 \times 10^{11}$ days.

$= 2.34 \times 10^9$ years

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5(d) Rotational kinetic Energy T is

$$\boxed{T = \frac{1}{2} I \omega^2}$$

$$\frac{dT}{dt} = \frac{1}{2} I \, 2 \vec{\omega} . \frac{d\vec{\omega}}{dt} = I \vec{\omega} . \vec{\alpha}$$

where $\vec{\omega}$ is angular velocity

$\vec{\alpha}$ is angular acceleration

Torque $\boxed{\vec{G} = I \vec{\alpha}}$

$$\Rightarrow \frac{dT}{dt} = I \vec{\alpha} . \vec{\omega} = \vec{G} . \vec{\omega}$$

$$\Rightarrow \boxed{\frac{dT}{dt} = \vec{G} . \vec{\omega}}$$

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i.e. change in kinetic energy per second or power delivered is Torque times angular velocity.

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5(e)

$p_e = \frac{1\ MeV}{c}$ Relativistic electron.

$\Rightarrow p_e c = 1\ MeV$

$E^2 = p^2c^2 + (m_0c^2)^2$

$m_0c^2 = 0.511\ MeV$ for electron.

$E^2 = (1)^2 + (0.511)^2$

$\Rightarrow \boxed{E = 1.1229\ MeV}$

Kinetic Energy of electron –

$E - m_0c^2 = 1.1229 - 0.511$

$\boxed{K.E. = 0.6119\ MeV}$

$p = mv = \frac{m_0}{\sqrt{1 - v^2/c^2}} \cdot v$

or $E = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}}$

$\Rightarrow 1.1229 = \frac{0.511}{\sqrt{1 - v^2/c^2}}$

$\Rightarrow \left(1 - \frac{v^2}{c^2}\right)^{1/2} = 0.455 \Rightarrow 1 - v^2/c^2 = 0.207$

$\boxed{v = 0.89c}$

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Q.6 a)

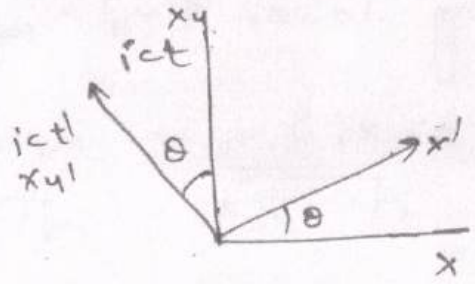
Lorentz transform

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$t' = \gamma(t - v/c^2 x)$



Minkowski space.

$x_1 = x_1' \cos\theta - x_4' \sin\theta$

$x_4 = x_1' \sin\theta + x_4' \cos\theta$

for $x' = 0 \Rightarrow \dfrac{x_1}{x_4} = \dfrac{-x_4' \sin\theta}{x_4' \cos\theta} = -\tan\theta$

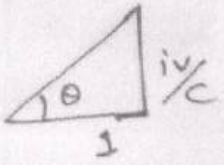
$\dfrac{x_1}{x_4} = -\tan\theta \Rightarrow \dfrac{x}{ict} = -\tan\theta$

$\Rightarrow \boxed{\dfrac{iv}{c} = \tan\theta}$ — (1)

Also

$$\begin{bmatrix} x_1' = x_1 \cos\theta + x_4 \sin\theta \\ x_4' = -x_1 \sin\theta + x_4 \cos\theta \end{bmatrix}$$

since $\tan\theta = \dfrac{iv}{c} \Rightarrow \sin\theta = \dfrac{iv/c}{\sqrt{1 - v^2/c^2}}$



$\cos\theta = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

putting in x_1' & x_4' equations —

$$x_1' = \frac{x_1}{\sqrt{1 - v^2/c^2}} + \frac{x_4 \, iv/c}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow x_1' = \gamma (x_1 + iv/c \, x_4)$$

$$x_1' = \gamma (x - vt)$$

Since
$x_1 = x$
$x_4 = ict$

Similarly

$$x_4' = \frac{-x_1 \, iv/c}{\sqrt{1 - v^2/c^2}} + \frac{x_4}{\sqrt{1 - v^2/c^2}}$$

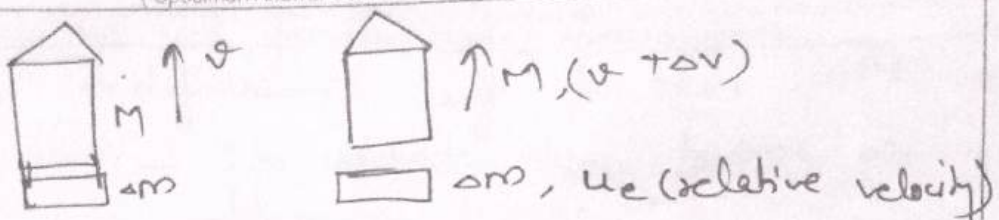
$$x_4' = \gamma \left(ict - \frac{iv}{c} x \right)$$

$x_4' = ict'$

$$\boxed{t' = \gamma (t - v/c^2 x)}$$

Thus Lorentz transformations can be defined as a rotation of axis through an imaginary angle.

Q6(b)



$$\vec{P}_i = (M + \Delta m)\vec{v}$$

$$\vec{P}_f = M(\vec{v} + \Delta\vec{v}) + \Delta m(-\vec{u}_e + \vec{v} + \Delta\vec{v})$$

$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

$$\Delta\vec{P} = M\Delta\vec{v} - \Delta m\,\vec{u}_e$$

$$\vec{F} = \frac{d\vec{P}}{dt} = M\frac{d\vec{v}}{dt} - \frac{dm}{dt}\vec{u}_e$$

rocket motion equation.

$$\vec{F} = M\vec{g}$$

$$M\vec{g} = M\frac{d\vec{v}}{dt} - \frac{dm}{dt}\vec{u}_e$$

In terms of rocket mass

$$M\vec{g} = M\frac{d\vec{v}}{dt} + \frac{dM}{dt}\vec{u}_e \quad \text{as } \frac{dM}{dt} = -\frac{dm}{dt}$$

$$\frac{d\vec{v}}{dt} = -\frac{1}{M}\frac{dM}{dt}u_e - g$$

as acceleration is directed downwards

$$\Rightarrow \boxed{v_f = v_0 + u_e \ln\frac{M_0}{M} - gt}$$

i> Min. rate of fuel consumption so rocket just takes off —

$\Rightarrow v_0 = v_f = 0$

$\Rightarrow u \ln \frac{M_0}{M} = gt$ $\boxed{M_0 = M - \alpha t}$

or rocket just balances gravitational force $\Rightarrow$

$Mg = -\frac{dm}{dt} u_e$

$\Rightarrow -\frac{dm}{dt} = \frac{Mg}{u_e}$

Mass $= M + m$
$= 120 kg$

(rate of fuel consumpt") $\frac{dm}{dt} = -\frac{(120) \times 9.8}{1.6 \times 10^3}$

$\boxed{\frac{dm}{dt} = -0.735 \text{ kg s}^{-1}}$

ii> final velocity when $\frac{dm}{dt} = 2 \text{ kg s}^{-1}$

$v_f = v_0 + u \ln \frac{M_0}{M} - gt$

final velocity when all fuel is Consumed $\Rightarrow M = 80 kg$ & $M_0 = 120 kg$ given $v_0 = 0$

$v_f = 1.6 \ln \frac{120}{20} - 9.8t$

$\Rightarrow v_f = 2\ln 6 - 9.8 \times 50$

also

$M_i = M_0 - qt$

$qt = 120 - 20$

$\Rightarrow t = \frac{100}{2} = 50s$

$v_f = 1600 \ln \frac{120}{20} - 9.8 \times 50$

$v_f = 2376.8 \ m s^{-1}$

for rate of fuel consumption –

$q = 20 \ kg/sec$

$t = \frac{120-20}{20} = 5 \ sec.$

$\Rightarrow v_f = 1600 \ln 6 - 9.8 \times 5$

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$v_f = 2817.8 \ m s^{-1}$

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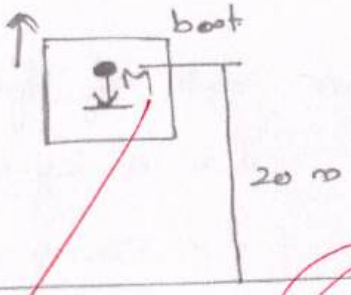
6(c)

If boy moves towards shore on boat, then boat also moves away from shore since no external forces are acting on system.

Thus momentum will be Conserved for the man-boat system.



Since no external torque/force ⇒ centre of mass will be unaffected.

⇒ $\boxed{M_1 \vec{r}_1 + M_2 \vec{r}_2 = 0}$ in Com frame.

⇒ $-10 \times 8 = 40 \times r_2$

⇒ $r_2 = 2$ m away from shore. (boat moves)

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Thus final location of boy with respect to shore is —

$20 - 8 + 2 = \boxed{16 \text{ m}}$